

Pensieve Header: The uBCH formula in the blobs quotient, perturbatively.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"];
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<< "Playground Program.m"
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{RR[1, 3], RR[2, 3]}
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$$\left\{ \frac{1}{2} \text{har}[1, 3] + \frac{1}{2} \text{har}[3, 1], \frac{1}{2} \text{har}[2, 3] + \frac{1}{2} \text{har}[3, 2] \right\}$$

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ToDegree[6, R[1, 3] ** R[2, 3]] // PCollect
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$$\begin{aligned} \text{ar}[2, 3] & \left(h + \frac{1}{2} h^2 h[1] + \frac{1}{12} h^3 h[1]^2 - \frac{1}{720} h^5 h[1]^4 - \right. \\ & \frac{1}{12} h^3 h[1] h[2] - \frac{1}{24} h^4 h[1]^2 h[2] - \frac{1}{180} h^5 h[1]^3 h[2] + \frac{h^6 h[1]^4 h[2]}{1440} + \\ & \left. \frac{1}{180} h^5 h[1]^2 h[2]^2 + \frac{1}{360} h^6 h[1]^3 h[2]^2 + \frac{1}{720} h^5 h[1] h[2]^3 + \frac{h^6 h[1]^2 h[2]^3}{1440} \right) + \\ \text{ar}[1, 3] & \left(h - \frac{1}{2} h^2 h[2] - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{720} h^5 h[1]^3 h[2] + \frac{1}{12} h^3 h[2]^2 + \right. \\ & \frac{1}{24} h^4 h[1] h[2]^2 + \frac{1}{180} h^5 h[1]^2 h[2]^2 - \frac{h^6 h[1]^3 h[2]^2}{1440} - \\ & \left. \frac{1}{180} h^5 h[1] h[2]^3 - \frac{1}{360} h^6 h[1]^2 h[2]^3 - \frac{1}{720} h^5 h[2]^4 - \frac{h^6 h[1] h[2]^4}{1440} \right) \end{aligned}$$

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ToDegree[6, R[3, 1] ** R[2, 3]] // PCollect
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$$\begin{aligned} \text{har}[2, 3] + \\ \text{ar}[3, 1] & \left(h + \frac{1}{2} h^2 h[2] + \frac{1}{12} h^3 h[2]^2 - \frac{1}{720} h^5 h[2]^4 + \frac{1}{12} h^3 h[2] h[3] + \frac{1}{24} h^4 h[2]^2 h[3] + \right. \\ & \frac{1}{180} h^5 h[2]^3 h[3] - \frac{h^6 h[2]^4 h[3]}{1440} + \frac{1}{180} h^5 h[2]^2 h[3]^2 + \\ & \left. \frac{1}{360} h^6 h[2]^3 h[3]^2 - \frac{1}{720} h^5 h[2] h[3]^3 - \frac{h^6 h[2]^2 h[3]^3}{1440} \right) + \\ \text{ar}[2, 1] & \left(-\frac{1}{2} h^2 h[3] - \frac{1}{12} h^3 h[2] h[3] + \frac{1}{720} h^5 h[2]^3 h[3] - \frac{1}{12} h^3 h[3]^2 - \right. \\ & \frac{1}{24} h^4 h[2] h[3]^2 - \frac{1}{180} h^5 h[2]^2 h[3]^2 + \frac{h^6 h[2]^3 h[3]^2}{1440} - \\ & \left. \frac{1}{180} h^5 h[2] h[3]^3 - \frac{1}{360} h^6 h[2]^2 h[3]^3 + \frac{1}{720} h^5 h[3]^4 + \frac{h^6 h[2] h[3]^4}{1440} \right) \end{aligned}$$

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ToDegree[6, R[3, 1] ** R[3, 2]] // PCollect
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har[3, 1] + har[3, 2]
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ToDegree[3, Expand[2 RR[1, 3]] ** Expand[2 RR[1, 3]]] // PCollect
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2 har[1, 3] + 2 har[3, 1]
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ToDegree[3, Expand[2 RR[1, 3]] ** Expand[2 RR[2, 3]]] // PCollect

$$\begin{aligned}
& -\frac{1}{3} h^3 \text{ar}[3, 3] h[1] h[2] + \frac{1}{6} h^3 \text{ar}[2, 2] h[1] h[3] + \frac{1}{6} h^3 \text{ar}[1, 1] h[2] h[3] + \\
& \text{ar}[3, 2] \left(h - \frac{1}{2} h^2 h[1] + \frac{1}{12} h^3 h[1]^2 - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{6} h^3 h[1] h[3] \right) + \\
& \text{ar}[2, 3] \left(h + \frac{1}{2} h^2 h[1] + \frac{1}{12} h^3 h[1]^2 - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{6} h^3 h[1] h[3] \right) + \\
& \text{ar}[1, 3] \left(h - \frac{1}{2} h^2 h[2] - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{12} h^3 h[2]^2 + \frac{1}{6} h^3 h[2] h[3] \right) + \\
& \text{ar}[3, 1] \left(h + \frac{1}{2} h^2 h[2] - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{12} h^3 h[2]^2 + \frac{1}{6} h^3 h[2] h[3] \right) + \\
& \text{ar}[2, 1] \left(-\frac{1}{2} h^2 h[3] - \frac{1}{12} h^3 h[1] h[3] - \frac{1}{12} h^3 h[2] h[3] - \frac{1}{6} h^3 h[3]^2 \right) + \\
& \text{ar}[1, 2] \left(\frac{1}{2} h^2 h[3] - \frac{1}{12} h^3 h[1] h[3] - \frac{1}{12} h^3 h[2] h[3] - \frac{1}{6} h^3 h[3]^2 \right)
\end{aligned}$$

ToDegree[6, Expand[2 RR[1, 3]] ** Expand[2 RR[2, 3]]] // PCollect

$$\begin{aligned}
& \text{ar}[3, 3] \left(-\frac{1}{3} h^3 h[1] h[2] + \frac{1}{120} h^5 h[1]^3 h[2] + \frac{1}{30} h^5 h[1]^2 h[2]^2 + \frac{1}{120} h^5 h[1] h[2]^3 - \right. \\
& \left. \frac{1}{60} h^5 h[1]^2 h[2] h[3] - \frac{1}{60} h^5 h[1] h[2]^2 h[3] - \frac{1}{60} h^5 h[1] h[2] h[3]^2 \right) + \\
& \text{ar}[2, 2] \left(\frac{1}{6} h^3 h[1] h[3] - \frac{1}{180} h^5 h[1]^3 h[3] - \frac{1}{60} h^5 h[1]^2 h[2] h[3] - \right. \\
& \left. \frac{1}{360} h^5 h[1] h[2]^2 h[3] + \frac{1}{180} h^5 h[1]^2 h[3]^2 + \frac{1}{90} h^5 h[1] h[2] h[3]^2 + \frac{1}{120} h^5 h[1] h[3]^3 \right) + \\
& \text{ar}[1, 1] \left(\frac{1}{6} h^3 h[2] h[3] - \frac{1}{360} h^5 h[1]^2 h[2] h[3] - \frac{1}{60} h^5 h[1] h[2]^2 h[3] - \right. \\
& \left. \frac{1}{180} h^5 h[2]^3 h[3] + \frac{1}{90} h^5 h[1] h[2] h[3]^2 + \frac{1}{180} h^5 h[2]^2 h[3]^2 + \frac{1}{120} h^5 h[2] h[3]^3 \right) + \\
& \text{ar}[3, 2] \left(h - \frac{1}{2} h^2 h[1] + \frac{1}{12} h^3 h[1]^2 - \frac{1}{720} h^5 h[1]^4 - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{24} h^4 h[1]^2 h[2] - \right. \\
& \frac{1}{180} h^5 h[1]^3 h[2] - \frac{h^6 h[1]^4 h[2]}{1440} + \frac{1}{180} h^5 h[1]^2 h[2]^2 - \frac{1}{360} h^6 h[1]^3 h[2]^2 + \\
& \frac{1}{720} h^5 h[1] h[2]^3 - \frac{h^6 h[1]^2 h[2]^3}{1440} + \frac{1}{6} h^3 h[1] h[3] - \frac{1}{24} h^4 h[1]^2 h[3] + \\
& \frac{1}{720} h^5 h[1]^3 h[3] + \frac{h^6 h[1]^4 h[3]}{1440} - \frac{1}{24} h^4 h[1] h[2] h[3] - \frac{1}{60} h^5 h[1]^2 h[2] h[3] + \\
& \frac{1}{96} h^6 h[1]^3 h[2] h[3] - \frac{7}{720} h^5 h[1] h[2]^2 h[3] + \frac{1}{96} h^6 h[1]^2 h[2]^2 h[3] + \\
& \frac{h^6 h[1] h[2]^3 h[3]}{1440} - \frac{1}{24} h^4 h[1] h[3]^2 + \frac{1}{80} h^5 h[1]^2 h[3]^2 - \frac{h^6 h[1]^3 h[3]^2}{1440} + \\
& \left. \frac{1}{240} h^5 h[1] h[2] h[3]^2 + \frac{1}{144} h^6 h[1]^2 h[2] h[3]^2 - \frac{h^6 h[1] h[2]^2 h[3]^2}{1440} + \right.
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{1}{120} h^5 h[1] h[3]^3 - \frac{1}{360} h^6 h[1]^2 h[3]^3 - \frac{1}{360} h^6 h[1] h[2] h[3]^3 - \frac{1}{720} h^6 h[1] h[3]^4 \right) + \\
 \text{ar}[2, 3] & \left(h + \frac{1}{2} h^2 h[1] + \frac{1}{12} h^3 h[1]^2 - \frac{1}{720} h^5 h[1]^4 - \frac{1}{12} h^3 h[1] h[2] - \frac{1}{24} h^4 h[1]^2 h[2] - \right. \\
 & \frac{1}{180} h^5 h[1]^3 h[2] + \frac{h^6 h[1]^4 h[2]}{1440} + \frac{1}{180} h^5 h[1]^2 h[2]^2 + \frac{1}{360} h^6 h[1]^3 h[2]^2 + \\
 & \frac{1}{720} h^5 h[1] h[2]^3 + \frac{h^6 h[1]^2 h[2]^3}{1440} + \frac{1}{6} h^3 h[1] h[3] + \frac{1}{24} h^4 h[1]^2 h[3] + \\
 & \frac{1}{720} h^5 h[1]^3 h[3] - \frac{h^6 h[1]^4 h[3]}{1440} + \frac{1}{24} h^4 h[1] h[2] h[3] - \frac{1}{60} h^5 h[1]^2 h[2] h[3] - \\
 & \frac{1}{96} h^6 h[1]^3 h[2] h[3] - \frac{7}{720} h^5 h[1] h[2]^2 h[3] - \frac{1}{96} h^6 h[1]^2 h[2]^2 h[3] - \\
 & \frac{h^6 h[1] h[2]^3 h[3]}{1440} + \frac{1}{24} h^4 h[1] h[3]^2 + \frac{1}{80} h^5 h[1]^2 h[3]^2 + \frac{h^6 h[1]^3 h[3]^2}{1440} + \\
 & \left. \frac{1}{240} h^5 h[1] h[2] h[3]^2 - \frac{1}{144} h^6 h[1]^2 h[2] h[3]^2 + \frac{h^6 h[1] h[2]^2 h[3]^2}{1440} \right) + \\
 & \left. \frac{1}{120} h^5 h[1] h[3]^3 + \frac{1}{360} h^6 h[1]^2 h[3]^3 + \frac{1}{360} h^6 h[1] h[2] h[3]^3 + \frac{1}{720} h^6 h[1] h[3]^4 \right) + \\
 \text{ar}[1, 3] & \left(h - \frac{1}{2} h^2 h[2] - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{720} h^5 h[1]^3 h[2] + \frac{1}{12} h^3 h[2]^2 + \right. \\
 & \frac{1}{24} h^4 h[1] h[2]^2 + \frac{1}{180} h^5 h[1]^2 h[2]^2 - \frac{h^6 h[1]^3 h[2]^2}{1440} - \frac{1}{180} h^5 h[1] h[2]^3 - \\
 & \frac{1}{360} h^6 h[1]^2 h[2]^3 - \frac{1}{720} h^5 h[2]^4 - \frac{h^6 h[1] h[2]^4}{1440} + \frac{1}{6} h^3 h[2] h[3] - \\
 & \frac{1}{24} h^4 h[1] h[2] h[3] - \frac{7}{720} h^5 h[1]^2 h[2] h[3] + \frac{h^6 h[1]^3 h[2] h[3]}{1440} - \\
 & \frac{1}{24} h^4 h[2]^2 h[3] - \frac{1}{60} h^5 h[1] h[2]^2 h[3] + \frac{1}{96} h^6 h[1]^2 h[2]^2 h[3] + \frac{1}{720} h^5 h[2]^3 h[3] + \\
 & \frac{1}{96} h^6 h[1] h[2]^3 h[3] + \frac{h^6 h[2]^4 h[3]}{1440} - \frac{1}{24} h^4 h[2] h[3]^2 + \frac{1}{240} h^5 h[1] h[2] h[3]^2 - \\
 & \frac{h^6 h[1]^2 h[2] h[3]^2}{1440} + \frac{1}{80} h^5 h[2]^2 h[3]^2 + \frac{1}{144} h^6 h[1] h[2]^2 h[3]^2 - \frac{h^6 h[2]^3 h[3]^2}{1440} + \\
 & \left. \frac{1}{120} h^5 h[2] h[3]^3 - \frac{1}{360} h^6 h[1] h[2] h[3]^3 - \frac{1}{360} h^6 h[2]^2 h[3]^3 - \frac{1}{720} h^6 h[2] h[3]^4 \right) + \\
 \text{ar}[3, 1] & \left(h + \frac{1}{2} h^2 h[2] - \frac{1}{12} h^3 h[1] h[2] + \frac{1}{720} h^5 h[1]^3 h[2] + \frac{1}{12} h^3 h[2]^2 - \right. \\
 & \frac{1}{24} h^4 h[1] h[2]^2 + \frac{1}{180} h^5 h[1]^2 h[2]^2 + \frac{h^6 h[1]^3 h[2]^2}{1440} - \frac{1}{180} h^5 h[1] h[2]^3 + \\
 & \frac{1}{360} h^6 h[1]^2 h[2]^3 - \frac{1}{720} h^5 h[2]^4 + \frac{h^6 h[1] h[2]^4}{1440} + \frac{1}{6} h^3 h[2] h[3] + \\
 & \left. \frac{1}{24} h^4 h[1] h[2] h[3] - \frac{7}{720} h^5 h[1]^2 h[2] h[3] - \frac{h^6 h[1]^3 h[2] h[3]}{1440} \right) +
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{24} h^4 h[2]^2 h[3] - \frac{1}{60} h^5 h[1] h[2]^2 h[3] - \frac{1}{96} h^6 h[1]^2 h[2]^2 h[3] + \frac{1}{720} h^5 h[2]^3 h[3] - \\
& \frac{1}{96} h^6 h[1] h[2]^3 h[3] - \frac{h^6 h[2]^4 h[3]}{1440} + \frac{1}{24} h^4 h[2] h[3]^2 + \frac{1}{240} h^5 h[1] h[2] h[3]^2 + \\
& \frac{h^6 h[1]^2 h[2] h[3]^2}{1440} + \frac{1}{80} h^5 h[2]^2 h[3]^2 - \frac{1}{144} h^6 h[1] h[2]^2 h[3]^2 + \frac{h^6 h[2]^3 h[3]^2}{1440} + \\
& \left. \frac{1}{120} h^5 h[2] h[3]^3 + \frac{1}{360} h^6 h[1] h[2] h[3]^3 + \frac{1}{360} h^6 h[2]^2 h[3]^3 + \frac{1}{720} h^6 h[2] h[3]^4 \right) + \\
\text{ar}[2, 1] & \left(-\frac{1}{2} h^2 h[3] - \frac{1}{12} h^3 h[1] h[3] + \frac{1}{720} h^5 h[1]^3 h[3] - \frac{1}{12} h^3 h[2] h[3] + \right. \\
& \frac{1}{24} h^4 h[1] h[2] h[3] + \frac{1}{90} h^5 h[1]^2 h[2] h[3] - \frac{h^6 h[1]^3 h[2] h[3]}{1440} + \\
& \frac{1}{90} h^5 h[1] h[2]^2 h[3] - \frac{1}{360} h^6 h[1]^2 h[2]^2 h[3] + \frac{1}{720} h^5 h[2]^3 h[3] - \\
& \frac{h^6 h[1] h[2]^3 h[3]}{1440} - \frac{1}{6} h^3 h[3]^2 - \frac{1}{24} h^4 h[1] h[3]^2 - \frac{1}{720} h^5 h[1]^2 h[3]^2 + \\
& \frac{h^6 h[1]^3 h[3]^2}{1440} - \frac{1}{24} h^4 h[2] h[3]^2 + \frac{1}{90} h^5 h[1] h[2] h[3]^2 + \frac{1}{96} h^6 h[1]^2 h[2] h[3]^2 - \\
& \frac{1}{720} h^5 h[2]^2 h[3]^2 + \frac{1}{96} h^6 h[1] h[2]^2 h[3]^2 + \frac{h^6 h[2]^3 h[3]^2}{1440} - \frac{1}{24} h^4 h[3]^3 - \\
& \frac{1}{80} h^5 h[1] h[3]^3 - \frac{h^6 h[1]^2 h[3]^3}{1440} - \frac{1}{80} h^5 h[2] h[3]^3 + \frac{1}{144} h^6 h[1] h[2] h[3]^3 - \\
& \left. \frac{h^6 h[2]^2 h[3]^3}{1440} - \frac{1}{120} h^5 h[3]^4 - \frac{1}{360} h^6 h[1] h[3]^4 - \frac{1}{360} h^6 h[2] h[3]^4 - \frac{1}{720} h^6 h[3]^5 \right) + \\
\text{ar}[1, 2] & \left(\frac{1}{2} h^2 h[3] - \frac{1}{12} h^3 h[1] h[3] + \frac{1}{720} h^5 h[1]^3 h[3] - \frac{1}{12} h^3 h[2] h[3] - \right. \\
& \frac{1}{24} h^4 h[1] h[2] h[3] + \frac{1}{90} h^5 h[1]^2 h[2] h[3] + \frac{h^6 h[1]^3 h[2] h[3]}{1440} + \\
& \frac{1}{90} h^5 h[1] h[2]^2 h[3] + \frac{1}{360} h^6 h[1]^2 h[2]^2 h[3] + \frac{1}{720} h^5 h[2]^3 h[3] + \\
& \frac{h^6 h[1] h[2]^3 h[3]}{1440} - \frac{1}{6} h^3 h[3]^2 + \frac{1}{24} h^4 h[1] h[3]^2 - \frac{1}{720} h^5 h[1]^2 h[3]^2 - \\
& \frac{h^6 h[1]^3 h[3]^2}{1440} + \frac{1}{24} h^4 h[2] h[3]^2 + \frac{1}{90} h^5 h[1] h[2] h[3]^2 - \frac{1}{96} h^6 h[1]^2 h[2] h[3]^2 - \\
& \frac{1}{720} h^5 h[2]^2 h[3]^2 - \frac{1}{96} h^6 h[1] h[2]^2 h[3]^2 - \frac{h^6 h[2]^3 h[3]^2}{1440} + \frac{1}{24} h^4 h[3]^3 - \\
& \frac{1}{80} h^5 h[1] h[3]^3 + \frac{h^6 h[1]^2 h[3]^3}{1440} - \frac{1}{80} h^5 h[2] h[3]^3 - \frac{1}{144} h^6 h[1] h[2] h[3]^3 + \\
& \left. \frac{h^6 h[2]^2 h[3]^3}{1440} - \frac{1}{120} h^5 h[3]^4 + \frac{1}{360} h^6 h[1] h[3]^4 + \frac{1}{360} h^6 h[2] h[3]^4 + \frac{1}{720} h^6 h[3]^5 \right)
\end{aligned}$$